JHEP HYPER VERSION

RECEIVED: *August 1, 2000*, ACCEPTED: *August 18, 2000*

Random values of the cosmological constant

John F. Donoghue

Department of Physics, University of Massachusetts
Amherst, Massachusetts 01003, and
TH Division, CERN, Geneva
E-mail: Donoghue@physics.umass.edu

ABSTRACT: One way that an anthropic selection mechanism may be manifest in a physical theory involves multiple domains in the universe with different values of the physical parameters. If this mechanism is to be relevant for understanding the small observed value of the cosmological constant, it may involve a mechanism by which some contributions to the cosmological constant can be fixed at a continuous range of values in the different domains. I study the properties of four possible mechanisms, including the possibility of the Hubble damping of a scalar field with an extremely flat potential. Another interesting possibility involves fixed random values of non-dynamical form fields, and a cosmological mechanism is suggested. This case raises the possibility of anthropic selection of other parameters in addition. Further requirements needed for a consistent cosmology are discussed.

KEYWORDS: Spontaneous Symmetry Breaking, Superstring Vacua, Cosmology of Theories beyond the SM, Physics of the Early Universe.

JHEP08(2000)022

Contents

1. Introduction

The problem of understanding a small but non-zero cosmological constant (Λ)[1] appears even harder than it would be if the cosmological constant were identically zero [2]. There are many contributions to Λ, ranging from zero-point energies to Higgs and QCD vacuum condensates. Observing a non-zero value tells us that we should not seek a principle that requires these contributions to cancel exactly. However, empirically a partial cancellation must occur and must be extremely fine-tuned in order to result in such a tiny residual.

The problem is so severe that it forces us to take seriously the anthropic [3] multiple domain solution which would naturally lead to the observation of a small non-zero Λ. Under this hypothesis [4]–[7], the cosmological constant is a parameter that can take on different values in different domains of the universe, with an assumed cosmological evolution such that we live entirely within a single domain. Domains with "normal" values of the cosmological constant would collapse quickly or expand exponentially rapidly and could not lead to life of any form. Only those with a small enough residual Λ have the conditions appropriate for life and it is only this restricted range that we should consider. Under this hypothesis, we would expect to observe a non-zero value of Λ, since there is no mechanism forcing it to be zero, and

[1] If the cosmological constant is indeed the explanation of the recent supernova observations [1], the value would be $\Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4$, where $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass.

the magnitude would be expected to be typical of the anthropically allowed range. Weinberg [4, 8] has phrased this constraint in a physical way by asking about the mean value of Λ in universes in which matter clumps into galaxies, the clumping being a needed precursor to life. Under plausible estimates [9], the observed value of Λ is reasonably typical of the mean viable value. Rees and Tegmark [10] have pointed out that in a more general context there is an allowed two dimensional area in the values of Q and Λ, where Q is the magnitude of the initial density perturbations, again such that our values are reasonably typical. While this hypothesis could provide a natural explanation of the value of Λ, its physical foundation remains unclear and we need to look for possible physical realizations.

For the mechanism to be contained in a physical theory there must be two main ingredients. The first is the generation of an appropriately large universe with domains that are presently disconnected. This is a relatively simple requirement. There are many available ideas for having quantum fluctuations or random dynamics influence physics within one causally connected region of the early universe. Inflation [6, 11], or pre-big-bang evolution [12] can then insure that we live entirely within a region which evolved from a single such domain. Disconnected regions of the universe are a common occurrence in modern theories of cosmology.

The difficult aspect of this hypothesis is contained in the second ingredient — the variability of physical parameters such as the cosmological constant. Ordinarily, coupling constants and masses are constant parameters uniquely defined within a theory. However in this hypothesis, the requirement is that these parameters can take on multiple values, yet are essentially constant throughout our domain. The values of the parameters are related to the ground state of the theory. Different ground states correspond to differences of at least some parameters of the low energy theory. The usual situation envisioned in fundamental theories is that there is a unique ground state to the theory, or at most a discrete few ground states. Even in string theory, which classically has continuous families of ground states, one normally expects that non-perturbative effects will select at most a few possible true ground states per compactification. However, it is unlikely that a set of discrete ground states is sufficient for implementing the anthropic selection (see the next section). If we turn to continuously variable states there are difficulties in maintaining a stable set of parameters. This paper examines issues associated with known mechanisms for implementing the multiple-domain/anthropic scenario.

2. Discrete versus continuous?

The ground state of a theory like QCD appears to be unique. The electroweak theory has a continuous family of ground states, corresponding to different directions of the Higgs field, but they are all equivalent and all have the same parameters. In more complicated theories with multiple Higgs fields there can be several minima to the

JHEP08(2000)022

Higgs potential. These multiple minima are potentially applicable to the multiple domain problem. One possibility is that some symmetry leads to the condition where several ground states have the same energy. However, if they have the same ground state energy then they have the same cosmological constant, and are therefore not useful in this context. In the more common case where the minima are all of different energy, only one will be the true ground state. However, in some situations the time to tunnel from one ground state to the true vacua can be long enough that the metastable states can be considered in cosmology. These different ground states would correspond to different cosmological constants. Therefore it is reasonable to consider multiple metastable discrete ground states as a candidate mechanism in the multiple domain problem.

However, a few discrete ground states are not enough for the anthropic solution for the cosmological constant. A theory with multiple ground states must occur at energies higher that that of the Standard Model. Let us denote the scale of this future theory by M_*, with $M_* \geq 1\,\mathrm{TeV}$. The ground states would be categorized by energies of this scale. In particular, the splitting between the ground state with the smallest negative cosmological constant and the smallest positive one would be of this size. It is extremely unlikely that a ground state would fall in the very tiny window that allows a anthropically viable cosmological constant. That window corresponds to a range

$$\frac{(\Delta\Lambda)_{\mathrm{anthropic}}}{(\Delta\Lambda)_{\mathrm{natural}}} \sim 10^{-58} \left[\frac{1\,\mathrm{TeV}^4}{M_*^4}\right]. \tag{2.1}$$

If a theory had a very densely packed set of states around $\Lambda = 0$, it would contain an unnaturally small parameter describing the spacing of these states (as well as possibly having difficulty arranging for these states to be metastable for long periods). The great disparity between M_*^4 and Λ indicates that one would require an additional mechanism to generate the possibility of fine tuning an anthropically acceptable value.[2]

The alternative is that the parameters can vary *continuously*, yet stay frozen at an arbitrary fixed value throughout our domain. Here the requirement is only that both signs of the cosmological constant be possible. In this case, random dynamics will occasionally generate an acceptable Λ in the neighborhood of zero. However, this option is not without problems.

Let us consider the basic difficulty in a general framework. If the parameters can have continuously different values, they would be different in causally disconnected domains in the early universe. Therefore they can be described by space-time dependent fields. This means that we will always be looking at the dynamics of some fields. Since by assumption these fields are not constrained to be near a unique

[2]In fact, new mechanisms that may allow closely spaced values for the cosmological constant have recently been addressed in [14, 15]

value, their potential, if they have any, must be small and they would normally be described as nearly-massless fields. While inflation can readily lead to these fields becoming uniform throughout the observed universe, the difficult part is to understand why the dynamics of such a field did not lead it to evolve towards a unique ground state. Therefore we are lead to consider fields whose dynamics have been frozen at continuous values in some fashion. This is the topic of the rest of this paper.

3. Hubble damping

There exists a simple mechanism that demonstrates that the freezing of dynamical fields at random values is indeed possible. It is related to the "slow roll" mechanism which is important for inflation.[3] Consider a scalar field in an expanding FRW universe governed by a scale factor $a(t)$. The equation of motion for this field is

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi = -V'(\phi)\,, \tag{3.1}$$

where a dot denotes a derivative with respect to time and the prime denotes differentiation with respect to the field ϕ. $V(\phi)$ is the potential for the scalar field and the Hubble parameter is defined by

$$H = \frac{\dot{a}(t)}{a(t)}\,. \tag{3.2}$$

For a field which is sufficiently spatially uniform, one can drop the term involving spatial gradients. In this case, a sufficiently flat potential, when compared to the Hubble constant, will lead to $\dot{\phi} \sim 0$. Thus the Hubble expansion can damp the time evolution of a uniform scalar field with a sufficiently flat potential. The application of this mechanism to the question of the anthropic solution to the cosmological constant problem has been studied in detail in recent work by Garriga and Vilenken [13], and discussed by Weinberg [8].

Let us look in more detail at the condition for the freezing of the field as we will see that there is a conflict related to the two uses of a flat potential in this scenario. On one hand the potential must be flat with respect the *present* Hubble parameter, which is a very small number, in order that the field be presently frozen. This corresponds to the intuitive expectation that the Hubble expansion plays very little role on the fields that we see around us, such that it must be a *very* weakly varying potential if the Hubble parameter is to provide the damping to keep the field from dynamically evolving. However, on the other hand, we need the potential to have

[3] However, it should be clearly stated that the discussion which follows does not apply to the field responsible for inflation, but to a different scalar field that is being invoked to address the cosmological constant problem.

enough variation that its contribution to the vacuum energy is sufficient to influence the cosmological constant. If the potential is too flat it contributes too weakly to the cosmological constant to be able to nearly cancel the other contributions to Λ. These dual requirements force certain unnatural conditions on the potential and also pose an important requirement on the nature of inflation.

The condition that the field remain effectively frozen today means, among other things, that it is not changing so fast as to contribute significantly to the present energy density. Thus its kinetic energy is bounded [13]

$$\frac{1}{2}\dot{\phi}^2 \ll \rho_0 \,, \tag{3.3}$$

where ρ_0 is the present energy density of the universe. This is related to the present Hubble constant (neglecting a possible curvature contribution to the Hubble constant) by

$$H_0^2 = \frac{8\pi}{3M_P^2}\rho_0 \,. \tag{3.4}$$

Using the slow roll approximation, we find that this kinetic constraint implies

$$V'(\phi) \sim 3H_0\dot{\phi} \ll 3H_0\sqrt{\frac{3M_P^2H_0^2}{8\pi}} \sim H_0^2 M_P \sim 10^{-122}M_P^3 \,. \tag{3.5}$$

Here we have use the kinetic energy bound, dropped constants of order unity and used the value of the Hubble parameter $H_0 \sim 10^{-61}M_P$. The conclusion is that the potential must be *very* flat. However, this means that reasonable variations of the magnitude of ϕ do not change the vacuum energy much. Specifically, for this mechanism to be operable we need to be able to nearly cancel the effect of other sources of vacuum energy, which we can denote by $\Lambda_{\rm other}$. The variation of the vacuum energy as we vary ϕ must then be of order $\Lambda_{\rm other}$. Let us distinguish two extreme situations: one where gravity or string theory provides the scale of the vacuum energy such that $\Lambda_{\rm other} \sim M_P^4$ and the other with low energy supersymmetry which implies $\Lambda_{\rm other} \sim 1\,{\rm TeV}^4$. In terms of the potential, the requirement is

$$V'(\phi)\Delta\phi \sim \Lambda_{\rm other} \,. \tag{3.6}$$

Thus the very small values of $V'(\phi)$ can only be useful if $\phi \sim \Delta\phi$ is very large. Inserting the constraint on $V'(\phi)$ from Hubble damping reveals just how large this value must be

$$\phi \gg 10^{122}M_P\left(\frac{\Lambda_{\rm other}}{M_P^4}\right) . \tag{3.7}$$

Even if we have low energy supersymmetry this leads to a strikingly large value of $\phi \gg 10^{58}M_P$.

The extreme flatness of the potential is a potential difficulty. Allowing quadratic and quartic couplings, we will have

$$V'(\phi) = \mu^2\phi + \lambda\phi^3 \,. \tag{3.8}$$

Given the constraints on ϕ and $V'(\phi)$, we must have $\mu^2 < 10^{-244} M_P^2$ and $\lambda < 10^{-488}$ in the case where $\Lambda_{\rm other}$ is determined by the Planck scale and $\mu^2 < 10^{-180} M_P^2$ and $\lambda < 10^{-296}$ in the most favorable case of weak scale supersymmetry breaking. At first sight this appears to be a fine tuning which is even greater than that of the cosmological constant. One might not be worried about the flatness of the potential since in supersymmetry flat potentials are ubiquitous, and one might hope that this flatness could be preserved. When supersymmetry is broken, radiative corrections will generate contributions to a potential. For a potential this flat, all matter fields certainly need to be decoupled from the ϕ field, or else they would generate a large potential after supersymmetry breaking. The decoupling of matter fields is also required in order to not violate general relativity constraints. The field ϕ is effectively massless, given the flatness of the potential, and would lead to observable long range forces if coupled to matter at even gravitational strength. However the constraints from the lack of radiative corrections to the potential are even stronger, and one is led to assume that matter fields can be completely decoupled from ϕ. This leads to the expectation that this field will not influence any of the other parameters of the Standard Model, as noted by Weinberg [8]. It will be a formidable problem to generate a potential that is large enough to influence the cosmological constant, yet flat enough to not be presently evolving. It would be remarkable if the existence of a viable domain is only possible due to the existence of such a extreme potential.

The needed initial conditions may also present a fine tuning problem. The size of the field ϕ is not by itself is not the problem, since we have seen that despite this large value the energy associated with the field is still below the Planck mass. However for a field of this size to also have its *kinetic* energy below the Planck scale requires an unnatural spatial and temporal constancy. In this case the problem is not so much in the present epoch, when inflation could have smoothed out any spatial variation in ϕ, but in the early universe before the start of inflation. At this time, in order that the kinetic energies not exceed the Planck scale, we need the variation in time and in each of the spatial directions to satisfy

$$\partial_0 \phi \sim \nabla_i \phi \sim \frac{\phi}{L} < M_P^2 \tag{3.9}$$

with L being the scale factor which describes the constancy of the field. (In an infinite universe L would be the wavelength of the field configuration.) If the scalar field evolves classically its magnitude would be the same in the early universe, and one is then constrained to have

$$\begin{aligned} L &> 10^{122} M_P^{-1} \sim 10^{80}\,\text{light-years} \qquad (\Lambda_{\rm other} \sim M_P) \\ &> 10^{58} M_P^{-1} \sim 10^{16}\,\text{light-years} \qquad (\Lambda_{\rm other} \sim 1\,\text{TeV}^4)\,. \end{aligned} \tag{3.10}$$

Thus the requirement is that the field initially (at the start of inflation) have an extremely large value, but have a incredibly tiny spatial and temporal variation. These initial values are quite unnatural, and tell us that the classical evolutions is not a natural solution.

However, quantum fluctuations during inflation can modify the field values, and if inflation is long enough, would remove the unnaturalness issue for the initial conditions.[4] This then can be converted into a limit on the length of the inflationary epoch. Quantum fluctuations behave differently in an exponentially expanding space time. Long wavelength modes can get redshifted such that they become almost flat, in which case the Hubble damping freezes them to constant values that add to the value of the classical field. Since the quantum fluctuations carry either sign, this leads to a random walk character for the net field values. Different regions in the inflating domain can thus develop different values of the field, with a rms deviation that grows as $\sqrt{t}$. The heuristic explanation is as follows, although the result is derived from more rigorous calculations [16, 17]. In each causally connected region of size H^{-1}, fluctuations are independent. (In this section H refers to the Hubble constant during the period of inflation, rather than in the present epoch.) In an expansion time of H^{-1} a typical fluctuation is of size $\Delta\phi \sim H/2\pi$. Since the expansion can freeze this field, over many expansion times these fluctuations then add as a random walk, resulting in a spread of values of order

$$\delta\phi^2 = \left(\frac{H}{2\pi}\right)^2 Ht\,. \tag{3.11}$$

Because these fluctuations take place during inflation, and the expansion smooths out the spatial variation, the kinetic energy constraint is never violated. As long as the potential energy does not grow larger than M_P, any value of the field can be reached in some domain if inflation goes on long enough. Thus if the initial value of ϕ starts off as $\phi \leq M_P$ and $H, \Lambda_{\rm other}$ are also of order M_P, one requires $N = Ht \sim 10^{244}$ e-foldings of inflation in order to have quantum fluctuations allow the field to grow to sufficient size to be relevant for the anthropic constraint. For the case where all of these quantities are as small as 1 TeV, the required number of e-foldings is 10^{148}. If a theory has only 60–100 e-foldings, the quantum fluctuations cannot solve the initial value problem. However the constraint on the amount of inflation can be solved naturally in the various versions of eternal inflation [17], in which inflating domains continue forever, and our domain can have undergone an unlimited amount of inflation. Therefore, the anthropic mechanism is most naturally embedded in theories of eternal inflation, as in [13].

The multiple domain hypothesis raises the possibility of naturally providing a way to solve the fine tuning problem. The Hubble damping mechanism is interest-

[4] I would like to thank Jaume Garriga and Alex Vilenkin for pointing out the importance of quantum fluctuations and the relevance of eternal inflation for the problem of the size of the field ϕ.

ing because it demonstrates that fields can become frozen at a continuous range of values. The difficulty with the extremely flat potential can be traced back to the the reliance on the Hubble term in the equation of motion to provide the mechanism for freezing the field. At present, H is too small to provide much influence on the behavior of fields. It is useful to search for more efficient methods of damping the dynamics.

4. Kinetic freezing

We could ask why, in the previous analysis, we could not simply redefine the scalar field by an overall scale such that its magnitude looks more normal, at the expense of also redefining parameters in the potential. The answer is that the condition that set the scale was the requirement that the kinetic terms be conventionally normalized. This suggests that by playing with the overall factor in front of the kinetic energy, one could also freeze the dynamics. In fact, this idea has been suggested in the context of hyperextended inflation [18], and a variant has recently been invoked to control the dilaton potential [19]. In this situation, one imagines that non-renormalizable interactions are present in the action, such that the lagrangian becomes

$$\mathcal{L} = \frac{1}{2} f(\phi, \psi)\partial_\mu \phi \partial^\mu \phi + V(\phi, \psi) + \cdots . \tag{4.1}$$

The function $f(\phi, \psi)$ is a unknown function that can depend on ϕ and on other fields, here labeled ψ. In this case, at values where f is large, the fields are effectively frozen even if fields are not at the minimum of the potential. This can be seen from the equation of motion for ϕ

$$f(\ddot{\phi} + 3H\dot{\phi}) + \frac{1}{2} f' \dot{\phi}^2 = -V'(\phi)\,, \tag{4.2}$$

where

$$f' = \frac{\partial f(\phi, \psi)}{\partial \phi}\,. \tag{4.3}$$

If f or f' is large, this can be a mechanism for slowing further dynamical evolution, yet it is problematic when applied to the cosmological constant.

The goal here is to allow a a more natural size of the potential. Using notation from section 3 this means that a potential of size

$$V'(\phi) \sim \frac{\Lambda_{\text{other}}}{M_P} \tag{4.4}$$

will allow $V'(\phi)\Delta\phi \sim \Lambda_{\text{other}}$ for ϕ ranging over a natural range $\Delta\phi \sim M_P$. (Here we will not worry about a few extra powers of ten). Since the smallest reasonable expectation for Λ_{other} is of order the scale of low energy supersymmetry, in the absence of other mechanisms, this means that we need a potential of rough size

$$V'(\phi) \sim \frac{1\,\text{TeV}^4}{M_P} \sim 10^{-64} M_P^3\,. \tag{4.5}$$

Combining the equation of motion with the constraint on $\dot{\phi}$, this means that we need

$$f' > 10^{58} M_P^{-1}\,. \tag{4.6}$$

While this mechanism may also be used to freeze the fields it is questionable whether it is reasonable to get non-renormalizable terms so large. In an effective field theory description, non-renormalizable terms occur as small corrections to the basic theory, due to interactions with degrees of freedom which are much heavier. The expectation of effective field theories, born out in known examples, is that once the non-renomrmalizable terms are of order unity, we excite the high energy degrees of freedom directly and the theory changes to a new effective theory in which these fields are dynamical variables. It is not natural to achieve such extremely large non-renormalizable interactions.

In fact, one can see that this is related to the mechanism of the previous section in the special situation where f either does not depend on other fields ψ, or these fields are held fixed at the minimum of a potential, $\psi = \langle\psi\rangle$, and an integrability constraint is satisfied. In this case a field redefinition changes the problem exactly back to the situation of the previous section. Define

$$\begin{aligned} \chi &= g(\phi)\,, \\ \partial_\mu \chi &= g'(\phi)\partial_\mu \phi\,. \end{aligned} \tag{4.7}$$

If we then identify

$$g'(\phi) = f^{1/2}(\phi, \langle\psi\rangle) \tag{4.8}$$

and this can be integrated to obtain g, the lagrangian is transformed into

$$\mathcal{L} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + V(g^{-1}(\chi)) + \cdots\,. \tag{4.9}$$

This is just a conventionally normalized action with a suppressed potential.

5. Radiative damping

Finally, what about other forms of damping? It is also possible to damp the motion of a field through the radiation of particles. In effect, a changing field can produce particles, which takes energy out of the field and hence slows down the rate of change. This effect has been studied in ref. [20] and is used in the theory of "warm inflation" [21] Let us consider the equations of motion

$$\ddot{\phi} + (\Gamma + 3H)\dot{\phi} = -V'(\phi) \tag{5.1}$$

with some unspecified damping Γ. This structure arises from the coupling of the field to other particles, with the radiation of the other particles damping the dynamics of the field. The proportionality of the damping to $\dot{\phi}$ is indicative that if the field is not changing it does not radiate. Perhaps this mechanism could lead to naturally frozen fields.

The difficulty in this case comes from the fact that Γ must be very small at present. Can we have $\Gamma\dot{\phi}$ as large as $10^{-64}M_P^3$ as required by the constraint of eq. (4.5)? The constraint on Γ comes from the generation of particles in the universe. The equation of motion is equivalent to the conservation of energy in a co-moving volume a^3

$$\frac{d}{dt}\left(a^3(t)\rho\right) = -p\frac{d}{dt}a^3 - \Gamma\dot{\phi}^2 \tag{5.2}$$

with energy density and pressure

$$\begin{aligned} \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \tag{5.3}$$

such that $\Gamma\dot{\phi}^2$ represents the rate of energy flow out of the field ϕ. Higher power dependence on $\dot{\phi}$ will not change our argument. A reasonably loose constraint on the rate of energy production is that it is smaller that the production of the full present energy in one Hubble time.

$$\Gamma\dot{\phi}^2 < H\rho_0 \sim 10^{-183}M_P^5\,. \tag{5.4}$$

This then lets us put a constraint on the damping term in the equation of motion using

$$\Gamma\dot{\phi} = \left(\Gamma\dot{\phi}^2\Gamma\right)^{1/2} < 10^{-91}M_P^3\left(\frac{\Gamma}{M_P}\right)^{1/2}\,. \tag{5.5}$$

Even if this unspecified damping mechanism was able to produce $\Gamma \sim M_P$, this fails by 27 orders of magnitude to provide enough damping to allow a reasonably sized potential.

6. Form fields

We may also turn to other ideas for fields with frozen dynamics. Another possibility is known in the supergravity literature, as first pointed out by [22, 23] Consider a field like a gauge potential but with three totally antisymmetric Lorentz indices

$$A_{\alpha\beta\gamma}(x) = -A_{\beta\alpha\gamma} = -A_{\gamma\beta\alpha} \tag{6.1}$$

such that its field strength tensor is also formed antisymmetrically

$$F_{\alpha\beta\gamma\delta} = \partial_{[\alpha} A_{\beta\gamma\delta]} \,, \tag{6.2}$$

where the square brackets denote the antisymmetrization of the indices. The Bianchi identity

$$\partial_{[\alpha} F_{\beta\gamma\delta\rho]} = 0 \tag{6.3}$$

is then always satisfied in 4 dimensions since there is no totally antisymmetric object with five Lorentz indices. The action

$$S_F = \frac{-1}{48} \int d^4x \sqrt{-g}\; F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} \tag{6.4}$$

leads to the equation of motion

$$\partial^\alpha \left[\sqrt{-g} F_{\alpha\beta\gamma\delta}\right] = 0 \,. \tag{6.5}$$

The only solution to this is

$$F_{\alpha\beta\gamma\delta} = \frac{c}{\sqrt{-g}} \epsilon_{\alpha\beta\gamma\delta} \tag{6.6}$$

for arbitrary c. Thus this field is non dynamical, with only a constant solution. Substitution of this solution in Einstein's equations shows that it behaves as a positive cosmological constant. In the language of differential forms, A is a 3-form potential, and F a 4-form field strength, with equations of motion and Bianchi identity

$$d * F_4 = 0 \,, \tag{6.7}$$

$$dF_4 = 0 \,. \tag{6.8}$$

Form fields appear in the low energy limit of string theory and M-theory. The most obvious is the type-II supergravity in the low energy limit of M-theory, where the 4-form field strengths occur explicitly. However, they can also be obtained by dimensional reduction from higher form fields. Consider a form field strength with more than four indices, $F_{\alpha\beta\gamma\delta\cdots\rho}$. Upon compactification, some of the indices can be assigned to the compact directions, becoming internal indices. The number of such 4-forms will depend on the particular number and symmetries of the compact subspaces. Four-forms may also appear from lower dimension forms. For an n-form in d dimensions, its dual is a $d-n$ form. Likewise duality relates a 4- form in 4-d to a zero-form -i.e. a constant.

Hawking and Turok [24] have proposed the generation of a non-zero 4-form through a tunneling mechanism involving an special instanton in the case of an open universe. This calculation remains controversial, with a dispute over the meaning of the instanton solution [25]. The mechanism has phenomenological problems, as it naturally predicts an almost empty universe. Moreover, the mechanism generates one value of Λ through out the entire universe, such that the naturalness of the anthropic selection is lost, and it does not correspond to the multiple domain structure under consideration here.

If a multiple domain structure is to be realized in nature, it will be generated in the early universe. Therefore we should look to cosmology for possible mechanisms. Here we suggest a mechanism which exploits the dimensional reduction that may take place in string theories The non-dynamical nature of the 4-form fields is only true in four dimensions. In higher dimensions, the equations of motion allow the usual plane wave solutions. The lack of dynamics in 4-d results from the restriction that the Lorentz indices and the space-time variability lie entirely within the 4-d space. This suggests a potential cosmological mechanism for the generation of the 4-form. Consider higher dimensional theories where compactification leads to a 4-d low energy theory. If cosmology goes through a phase where fields above the compactification scale are excited at some time in the early universe, 4-form fields will be dynamical. They will have fluctuating values, with a non-zero rms field strength. As the universe expands and the average energy decreases, the Kaluza Klein modes with excitations in the compact dimensions will decouple leaving an effective four dimensional theory. As this transition occurs, the 4-form fields will become non dynamical and will be frozen into random values in different space-time regions. As the universe evolves to lower energies, these values remain frozen. When supplemented by inflation, such that we see only the field from a very small initial patch, this can result in the multiple domain scenario.

In string theory there appears to be a barrier to the use of form fields to generate random values of Λ. In a string theory ground state, the values of the form field strengths are quantized [14, 26, 27]. This occurs because there are both electric and magnetic charges coupled to the form fields. By analogy to the usual electric and magnetic charges, these charges are quantized. Construction of various gaussian surfaces then imply that the flux, and hence the magnitudes of the constant form fields, are also quantized. The cosmological mechanism described above could also generate different values of the quantized form fields, but it might appear that unless the size of the quanta are extremely small, the likelihood of solving the cosmological constant problem is small.[5]

However the quantization constraint can still allow the form fields to take on all values in a continuous range providing other fields adjust accordingly. The quantization constraint involves V_7, the volume of the compact seven dimensional manifold [14]. There are also additive contributions from possible flat background gauge potentials [28] and constant fermion densities.[6] If these were all to attain their low-energy values first, then the form field condensate would be forced to certain discrete values. However, in the early universe the moduli controlling V_7, the gauge potentials and the fermions are fluctuating. The form field can take on any continuous value as long as the other fields are adjusted to values consistent with the quantiza-

[5]Of course it is also possible to imagine form fields without invoking string theory. It has also been argued that there can be form fields which are not coupled to string theory charges [29].

[6]This can be seen from the supergravity equations of motion.

tion constraint. As the universe cools to lower energies, the form field will become non dynamical and will stay at its constant value. At low energies the potentials for the moduli and other fields will become important and will approach their zero-temperature form. These fields will then seek the minimum of their potentials, with the quantization constraint being a constraint on what values are possible. On other words, the form field value will become a constraint on vacuum selection because it is no longer able to evolve. This inverts the usual reasoning, with the result that the form fields could end up at any value but the vacuum state adjusts in order to satisfy the quantization condition.

The frozen fields will have two effects. First, they can contribute directly to the cosmological constant. However, there is also an indirect secondary effect through the dilaton and moduli fields. As emphasized in ref. [30], the form fields which carry string theory charges can influence the potentials for the moduli and dilaton fields. The moduli and dilaton potentials vanish perturbatively, yet it is expected that non-perturbative effects will generate potentials for these fields. The frozen background of form fields will give additive contributions to the potentials. This would amount to random shifts in the moduli potentials in different domains, and would influence the ground state solution and the parameters of the low energy theory. This will then provide a further shift in the ultimate cosmological constant, since every mass and coupling contributes to some extent to the vacuum energy. The influence on the moduli values may lead to the expectation that other parameters in the theory also are variable.

In general, non-zero values of the form fields break supersymmetry. It is known that there are special combinations of compactification and vacuum expectation values that allow the existence of low-energy supersymmetry [30, 31]. Whether it is natural that such special situations occur in the early universe is an open question. However, it may even be preferable that the supersymmetry is broken at high scales (depending ultimately on the outcome of future experiments, of course). In theories with random coupling constants, the fine tuning problem of the Higgs vev may not be the most serious issue. As with the cosmological constant, there is a plausible anthropic constraint such that we would only live in regions with a small Higgs vev [32]. This occurs because if the vev is much larger than observed, the elements other than hydrogen do not exist and we lack the complexity needed for life. The variability of the form fields and the moduli could allow the realization of this anthropic constraint also. Moreover, low energy supersymmetry poses significant problems for cosmology. Scalar particles with TeV scale masses are ubiquitous in such theories, and the dilaton in particular is model independent. These particles dominate the energy density of the universe for so long that they spoils nucleosynthesis [34]. This problem, a string variant of the Polonyi problem, has proven difficult to overcome. Moreover, it appears difficult to implement inflation in theories with low energy supergravity [36]. So cosmology may welcome the situation where supersymmetry is broken at a high scale.

JHEP08(2000)022

Let us then summarize the ingredients of a cosmology that would make use of this mechanism. The first obvious requirement is that the evolution of the universe must involve an early period where energies above the compactification scale are excited. This is needed in order to excite the form fields. The required features of compactification has not yet been studied much because most analyses have been done under the assumption that supersymmetry survives to low energy. So we donot yet know the full possibility for the field content below the scale of non-supersymmetric compactifications. However, the supersymmetric spectrum above the compactification scale has many fields, the dilaton and moduli, that have the possibility of playing the role of the inflaton [35]. Use of these fields would likely be possible if inflation and compactification occur at the same scale. Finally we clearly need sufficient inflation to smooth out any initial gradients in the fields.

7. Summary

This paper is a preliminary investigation into the field theory dynamics that could lead to continuous random contributions to the cosmological constant in theories with multiple domains with different parameters. Damping mechanisms appear to require rather extreme values for the potentials, the fields and/or the non-renormalizable interactions. As noted by Weinberg [8], the need to decouple all other fields from this scalar field, in order to preserve the flatness of the potential, has the consequence that it will not influence other parameters in the theory — that the cosmological constant will be the only parameter for which an anthropic constraint is relevant

However 4-form fields appear as a quite natural mechanism. For this to be applicable, we would want an energetic initial condition, to excite the form fields, and a inflationary phase to generate the uniformity of the observed universe. The fact that the form fields also influence the dilaton and moduli fields of string theory is also interesting. This would generate a chaotic component to the vacuum selection procedure and would thus influence the other parameters in the theory also. This may then also for the Higgs vev fine-tuning problem. There exists the possibility of testing the distribution of some of the parameters through the weight of the quark mass distribution [37]. It remains to be seen whether a fully complete model along these lines may be developed.

This paper has explored the situation in which the field variables influencing the cosmological constant are continuous. In this situation it is quite natural that the cosmological constant should occasionally be close enough to zero to satisfy Weinberg's anthropic constraint. In a recent paper, Bousso and Polchinski [14] have addressed the situation where multiple form fields can plausibly lead to discrete but closely spaced values for the cosmological constant appropriate for an anthropic selection. The spacing of the values with separation of order $10^{-122}M_P^4$ appears to require very large internal dimensions or very many (of order 100) form fields.

Acknowledgments

I would like to thank David Kastor, Renata Kallosh, Andrei Linde, Joe Polchinski and especially Daniel Waldram, Jaume Garriga and Alexander Vilenkin for useful conversations. I also thank CERN for their kind hospitality during most of time devoted to this work. This work has been supported in part by the U.S. National Science Foundation and by the John Templeton Foundation.

References

[1] Supernova Cosmology Project collaboration, *Measurements of omega and lambda from 42 high-redshift supernovae*, *Astrophys. J.* **517** (1999) 565 [astro-ph/9812133];
A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, *Astron. J.* **116** (1998) 1009 [astro-ph/9805201].

[2] S. Weinberg, *The cosmological constant problem*, *Rev. Mod. Phys.* **61** (1989) 1;
S.M. Carroll, *The cosmological constant*, astro-ph/0004075.

[3] J.D. Barrow and F. Tipler, *The anthropic cosmological principle*, Clarendon Press, Oxford 1986;
C.J. Hogan, *Why the universe is just so*, astro-ph/9909295.

[4] S. Weinberg, *Theories of the cosmological constant*, astro-ph/9610044.

[5] A. Linde, *Inflation and quantum cosmology*, in *Three hundred years of gravitation*, S.W. Hawking and W. Israel eds., Cambridge University Press, Cambridge 1987, p. 604

[6] A. Linde, *Particle physics and inflationary cosmology*, Harwood, Chur 1990,

[7] A. Vilenkin, *Predictions from quantum cosmology*, *Phys. Rev. Lett.* **74** (1995) 846 [gr-qc/9406010].

[8] S. Weinberg, *The cosmological constant problems*, astro-ph/0005265.

[9] H. Martel, P.R. Shapiro and S. Weinberg, *Likely values of the cosmological constant*, astro-ph/9701099.

[10] M. Tegmark and M.J. Rees, *Why is the CMB fluctuation level* 10^{-5}*?*, *Astrophys. J.* **499** (1998) 526 [astro-ph/9709058].

[11] A. Linde, *Lectures on inflationary cosmology*, hep-th/9410082.

[12] M. Gasperini and G. Veneziano, *Pre-Big Bang in string cosmology*, *Astropart. Phys.* **1** (1993) 317 [hep-th/9211021].

[13] J. Garriga and A. Vilenkin, *On likely values of the cosmological constant*, *Phys. Rev.* **D 61** (2000) 083502 [astro-ph/9908115].

[14] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, *J. High Energy Phys.* **06** (2000) 006 [hep-th/0004134].

[15] J.L. Feng, J. March-Russell, S. Sethi and F. Wilczek, *Saltatory relaxation of the cosmological constant*, hep-th/0005276.

[16] A. Vilenkin and L.H. Ford, *Gravitational effects upon cosmological phase transitions*, *Phys. Rev.* **D 26** (1982) 1231;
A.D. Linde, *Scalar field fluctuations in expanding universe and the new inflationary universe scenario*, *Phys. Lett.* **B 116** (1982) 335;
A.A. Starobinsky, *Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations*, *Phys. Lett.* **B 117** (1982) 175;
A.H. Guth and S.-Y. Pi, *The quantum mechanics of the scalar field in the new inflationary universe*, *Phys. Rev.* **D 32** (1985) 1899.

[17] P.J. Steinhardt, *Natural inflation, Invited talk given at Nuffield workshop on the very early universe*, Cambridge, 1982;
A. Vilenkin, *The birth of inflationary universes*, *Phys. Rev.* **D 27** (1983) 2848;
A.D. Linde, *Eternally existing selfreproducing chaotic inflationary universe*, *Phys. Lett.* **B 175** (1986) 395;
A.H. Guth, *Inflation and eternal inflation*, *Phys. Rep.* **333** (2000) 555 [astro-ph/0002156].

[18] P.J. Steinhardt and F.S. Accetta, *Hyperextended inflation*, *Phys. Rev. Lett.* **64** (1990) 2740;
P.J. Steinhardt, *General considerations of the cosmological constant and the stabilization of moduli in the brane-world picture*, *Phys. Lett.* **B 462** (1999) 41 [hep-th/9907080].

[19] G. Huey, P.J. Steinhardt, B.A. Ovrut and D. Waldram, *A cosmological mechanism for stabilizing moduli*, *Phys. Lett.* **B 476** (2000) 379 [hep-th/0001112].

[20] A. Albrecht, P.J. Steinhardt, M.S. Turner and F. Wilczek, *Reheating an inflationary universe*, *Phys. Rev. Lett.* **48** (1982) 1437.

[21] A. Berera, *Warm inflation*, *Phys. Rev. Lett.* **75** (1995) 3218 [astro-ph/9509049];
A. Berera, M. Gleiser and R.O. Ramos, *A first principles warm inflation model that solves the cosmological horizon/flatness problems*, *Phys. Rev. Lett.* **83** (1999) 264 [hep-ph/9809583].

[22] A. Aurilia, H. Nicolai and P.K. Townsend, *Hidden constants: the theta parameter of QCD and the cosmological constant of $N = 8$ supergravity*, *Nucl. Phys.* **B 176** (1980) 509.

[23] M.J. Duff and P. van Nieuwenhuizen, *Quantum inequivalence of different field representations*, *Phys. Lett.* **B 94** (1980) 179;
M.J. Duff, *The cosmological constant in quantum gravity and supergravity*, presented

at *2nd Oxford quantum gravity conf.*, April 1980, based on earlier talk given at Europhysics Conf., *Unification of fundamental interactions*, Erice, Sicily, Mar 17-24, 1980.

[24] N. Turok and S.W. Hawking, *Open inflation, the four form and the cosmological constant*, *Phys. Lett.* **B 432** (1998) 271 [hep-th/9803156].

[25] A. Vilenkin, *The quantum cosmology debate*, gr-qc/9812027.

[26] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017]; *String duality: a colloquium*, *Rev. Mod. Phys.* **68** (1996) 1245 [hep-th/9607050]; *Tasi lectures on D-branes*, hep-th/9611050.

[27] J. Polchinski and A. Strominger, *New vacua for type-II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227].

[28] E. Witten, cited in ref. [14].

[29] B. Bajc and G. Gabadadze, *Localization of matter and cosmological constant on a brane in anti de Sitter space*, *Phys. Lett.* **B 474** (2000) 282 [hep-th/9912232].

[30] A. Lukas, B.A. Ovrut and D. Waldram, *Stabilizing dilaton and moduli vacua in string and M-theory cosmology*, *Nucl. Phys.* **B 509** (1998) 169 [hep-th/9611204].

[31] K.A. Meissner and M. Olechowski, *Towards supersymmetric cosmology in M-theory*, *Phys. Lett.* **B 480** (2000) 312 [hep-th/9910161].

[32] V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, *Anthropic considerations in multiple-domain theories and the scale of electroweak symmetry breaking*, *Phys. Rev. Lett.* **80** (1998) 1822 [hep-ph/9801253];
V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, *The anthropic principle and the mass scale of the standard model*, *Phys. Rev.* **D 57** (1998) 5480 [hep-ph/9707380].

[33] T. Banks, M. Berkooz and P.J. Steinhardt, *The cosmological moduli problem, supersymmetry breaking and stability in postinflationary cosmology*, *Phys. Rev.* **D 52** (1995) 705 [hep-th/9501053].

[34] B. de Carlos, J.A. Casas, F. Quevedo and E. Roulet, *Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings*, *Phys. Lett.* **B 318** (1993) 447 [hep-ph/9308325];
T. Banks, D.B. Kaplan and A.E. Nelson, *Cosmological implications of dynamical supersymmetry breaking*, *Phys. Rev.* **D 49** (1994) 779 [hep-ph/9308292].

[35] T. Banks, M. Berkooz, S.H. Shenker, G. Moore and P.J. Steinhardt, *Modular cosmology*, *Phys. Rev.* **D 52** (1995) 3548 [hep-th/9503114];
T. Banks, *M-theory and cosmology*, hep-th/9911067.

[36] A. Linde and A. Riotto, *Hybrid inflation in supergravity*, *Phys. Rev.* **D 56** (1997) 1841 [hep-ph/9703209].

[37] J.F. Donoghue, *The weight for random quark masses*, *Phys. Rev.* **D 57** (1998) 5499 [hep-ph/9712333].